A DEVELOPMENT MODEL OF FUZZY GOAL PROGRAMMING FOR REGIONAL RIVER WATER QUALITY MANAGEMENT

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Abstract

This paper proposes a development of Fuzzy Goal Programming model with imprecise information goal for optimizing the regional water quality management. Imprecision is associated with fuzziness which is non-statistical in nature and refers to the absence of sharp boundary in the information. Imprecise standards in environmental management objective could be represented by fuzziness functions. The objectives of management to be achieved in this model are categorized as: (i) Environmental objectives i.e. to maintain an ambient water quality in two parameters and standards, BOD(Biochemical Oxygen Demand) and DO (Dissolved Oxygen) closed to desired stream standard goal and to achieve an improvement sanitation program target;(ii) Economic objective i.e. to minimize the total cost of removing residual by an alternatives waste of treatment systems. The model of "Streeter Phelps" is adopted to represent the ecological relationship between organic pollutant loading and fluctuated water quality. Then fuzzy goal programming is developed as one of Multi Criteria Decision Making (MCDM) approach to "optimize" the objectives of decisions, in order to calculate how many pollutant should be removed at each system. The solution of the model provides a way of trade off analysis between attainment of objectives and useful imprecise information to the decision maker of environmental planning

Key words: Multi Criteria Decision Making (MCDM), Fuzzy Goal Programming, Water Quality Management, Pollution Modelling

1. INTRODUCTION

Uncertainty plays an important role in most water quality management problems. The random character of the natural processes governing water resources, the estimation errors in parameters of water quality models and the vagueness of planning objectives and constraints are all possible sources of uncertainty (Karmakar and Mujumdar, 2006). These potential sources uncertainty can significantly affect decision-making in water quality management. According to Julien (1994) and Karmakar and Mujumdar (2006) probability theory might be appropriate for dealing with only one specific type of uncertainty and other approaches have been developed to complement it.

The fuzzy set theory offers a framework to represent imprecise information. Imprecision expresses the absence of sharp boundaries and exactness in some information, while randomness refers to the uncertainty about the occurrence of an event (Tiwari, 1987 and Julien, 1994).

Decisions in the water quality management are often made in the basis of imprecise information. Goals setting of desired stream standards or constraints may not be defined precisely due to ill-defined and subjective requirements based on human judgments or preferences. In fact, standards are often set based on imprecise environmental goals and their determination requires risk assessments which are influenced by subjective and imprecise value judgment.

Previous works used optimization techniques in single objective function and without considering the imprecise information used in the setting objectives (Beck, 1987, Joanaz and Camara, 1994). This paper proposes Fuzzy Goal Programming model development with imprecise information goal for optimizing the regional water quality management.

2. FUZZY GOAL PRORAMMING MODEL

The use of fuzzy set theory in goal programming was first discussed by Hannan (1981) and Ignizio (1982). Tiwari *et al.* (1987 and 1986) presented various aspects of decision problem using fuzzy goal programming. The application of fuzzy goal programming in real world decision are found in numerous publication (Ciptomulyono, 2000). An approach to solve the fuzzy goal programming procedures have been developed. The typical such model could be defined as:

Find X

to satisfy $G_s(X) \geq g_s$, s = 1, 2, . . ., m (1)

Subject to

$$AX \geq 0 \quad (2)$$
$$X \geq 0$$

where X is an vector of decision variables and $AX \leq b$ are system constraints. Let's consider $G_s(X) \geq g_s$ or $G_{is}(X) \leq g_s$ as imprecise forms (fuzzy) of the aspiration level. According to Zimmermann (1988) a linear membership function (μ_s) for the each fuzzy goals could be expressed as follows:

$$\text{Maximize } V(\mu) = \sum_{s=1} \mu_s \quad (3)$$

Subject to

$$\mu_s = \frac{G_s(X) - L_s}{g_s - L_s} \quad (4)$$
$$AX \leq b \quad (5)$$
$$\mu_s \leq 1 \quad (6)$$
$$X, \mu_s \geq 0, \; s = 1, 2, \ldots, m \quad (7)$$

where L_s is the lower tolerance limit for the fuzzy goal G_s (X).and U_s is the upper tolerance limit. The criterion of the objective is to maximize membership function of decision (μ_s) instead of minimizing the deviation.

The Development Model of Water River Quality Management

The well known "Streeter Phelps" equation is quite useful for describing the behaviour of the DO fluctuation due to discharging a high concentration of organic pollutant (BOD) in the body of water. At the stream receiving a biodegradable waste, the BOD is upset and DO drops to lower level. The re-aeration process is indicated as first order reaction and is primarily related to the degree turbulence and natural mixing in the water.

Ciptomulyono (1992) approached the modelling as follows: the stream is divided into m zones where the i`th is defined as portion of stream between the i' th and (i+1) point of discharge. In each zone, all discharging waste is assumed to concentrate and to aggregate at the beginning of each zone, monitored at the lower end of each zone, before discharging waste at the next subsequent lower zone. Assuming that the stream system is under steady state condition in single dimension analysis, which longitudinal and vertical dispersion factors are neglected. There is a concentrated discharge at the beginning of the zone and the results of the biological processes then are monitored at the lower end of each zone.

Let define L_i is a BOD concentration at the beginning zone before any discharging of aggregated organic pollutant load (in kg BOD_5 per day). Then L_i denotes BOD level after the oxida-tion process at the lower end of zone i. Based on the material balance and the above assumptions, a linear expression between node i and node (i + 1) at the lower end of the zone i may be written as:

$$L_i = \hat{L}_i + L(BOD_i) \quad (8)$$

$L(BOD_i)$ represents an additional concentration of BOD associated with discharging of pollutant load (kg/day).It is presumably a proportional function of grouping pollutant load BOD_i in zone i and average debit of stream R_i. Introduce a factor β to change units of parameter. An estimation of concentration BOD at the beginning of the zone-i can be obtained as follows:

$$L_i = \hat{L}_{i-1} + \frac{\beta BOD_i}{R_i} \quad (9)$$

By considering the BOD first order reaction, the concentration estimation after reaching the lower end of each zone could be estimated as follows:

$$\hat{L}_i = (\hat{L}_{i-1} + \frac{\beta BOD_i}{R_i}) e^{-k_1^i \alpha \frac{d_i}{v_i}} \quad (10)$$

Where:

d_i = length of zone-i

V = average velocity of stream in zone i

k_1^i = constant rate of BOD reaction in zone i

α = a factor to convert the units of relevant parameter

As long as the relationship between pollutant load at the beginning of zone i and the quality of DO and BOD in the lower end ($\hat{D}_i$ and $\hat{L}_i$) follows a simple "Streeter Phelps" for zone i = 1, ..., m could be written as in equation (11)

The steady state assumption (under average condition for each zone) allows the DO reduction and BOD increase due to effluent discharge subsequently as constant coefficient transfer a_{ij} and b_{ij}. Where a_{ij} denotes a unit deficit of DO and b_{ij} is increasing BOD concentration in zone i due to pollutant discharge in zone j. To obtain these values, a little manipulation from equations (13) (12) could be approximated by assuming that both equations are differentiable to BOD variable.

$$\hat{D}_i = \frac{k_1^i}{k_2^i - k_1^i}\left(e^{-k_1^i \alpha \frac{d_i}{v_i}} - e^{-k_2^i \alpha \frac{d_i}{v_i}}\right)\hat{L}_i + \left(e^{-k_2^i \alpha \frac{d_i}{v_i}}\right)\hat{D}_{i-1}$$

Where:

$$\hat{L}_i = L_i\, e^{-k_1^i \alpha \frac{d_i}{v_i}} \;\; and \;\; L_i = \hat{L}_{i-1} + \frac{\beta\, BOD_i}{R_i} \qquad (11)$$

i = 1, 2 , 3, . . m (number of zone in the stream)

$\hat{D}_0$ = the DO deficit before entering the pollutant load at zone 1 and assumed to be obtainable.

k_2^i = the constant reoxygenation rate in zone i.

Given NPS_i^{DO} as the non point DO source in zone i and NPS_i^{BOD} is the non point BOD source in zone i. The BOD and DO parts of the model are subject to desired standards ($STDBOD_i$ and $STDO_i$) for each zone could be formulated by a system of linear inequalities as follows:

- For achievement desired standard of DO:

$$C_i^s - \left(\sum_i a_{ij} X_j + NPS_i^{DO}\right) \geq STDO_i \qquad (12)$$

for all j and i ≥ j

where:

C_i^s = the saturation of DO in zone i,

X_j = vector decision variables that represent an effluent discharge

- For achievement desired standard of BOD:

$$\sum_i \left(a_{ij} X_j + NPS_i^{BOD}\right) \leq STBOD_i \qquad (13)$$

Structure of Decision Model.

Considering that the strategy and policy to achieve an desired water quality standard of management in this case will be designed in the following system: (i) There are two controllable pollutant sources being considered and focusing on point source of pollutant only, i.e. industrial waste and domestic residual no improvement in non point source of pollutant; (ii) Considering industries limited to plants which discharge the organic pollutant, for the remaining industries no discharge BOD is assumed; (iii) There will be 3 systems of waste water designed to remove the pollutant load: a conventional primary treatment (PT), a secondary treatment (ST) for pollutant removal in each plant before discharging into the river, the third system is a collective system (CT).

Decision Variables

Based on given strategy policy, decision variables can be developed namely as an amount of organic pollutant that must be treated to the various alternative of treatment system for each zone. The structure of decision model related to strategy of removal pollutant could be described (Figure 1)

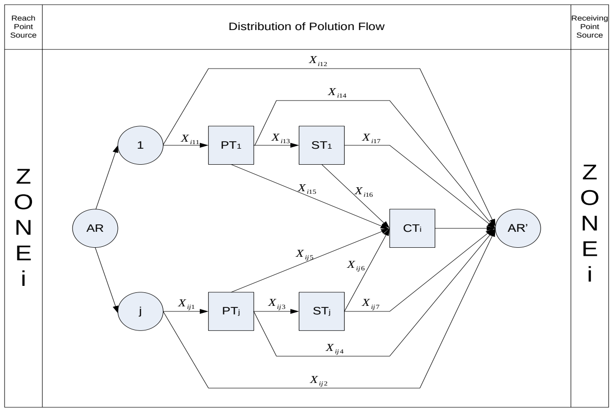

Figure 1. Structure of Decision Model

The decision variable of X_{ijk} is defined as the amount of industrial pollutant from source -j at zone i allocated to alternative treatment system k.

Goal Constraints and Objective Functions

a. Environmental Quality Goals

• The DO Standards.

Referring to the main objective of environmental quality management, minimum DO value has to be considered at the first priority objective. Thus, by little manipulation from equation (12), goal constraint function for m zones may be expressed mathematically as follows:

$$\sum_{i=1}^{m} a_{ri}\left\{\sum_{j=1}^{q_i}\sum_{k\in S_1} X_{ijk} + (1-\eta_i^c)\sum_{j=1}^{q_i}\sum_{k\in S_2} X_{ijk}\right\} \overset{\frown}{<} C_i^s - STDO_i - NPS_i^{DO} \qquad (14)$$

for r = 1, 2,….., m. And $r > m$, $a_{ri} = 0$

Where:

S_1 = the index of set decision variables that discharge the pollutant and effluent to the receiving point of stream directly

S_2 = the index of set decision variables that discharge the pollutant to the collective treatment system.
i = number of zones in the stream and q_i total number of industries in zone i.

- The Standards of BOD .

These goal constraints concern to minimize BOD levels in each zone, which must not exceed the maximum allowable concentration as determined by the STDBOD. Developed from equation (13), goal constraint functions could be expressed as follows:

$$\sum_{i=1}^{m} b_{ri} \left\{ \sum_{j=1}^{q_i} \sum_{k \in S_1} X_{ijk} + (1-\eta_i^c) \sum_{j=1}^{q_i} \sum_{k \in S_2} X_{ijk} \right\} \widetilde{<} STBOD_i - NPS_i^{BOD} \quad (15$$

for r = 1, 2, . . . , m, and $r > m$, $b_{ri} = 0$.

b. Economic Objective

The economic objective of this model is only represented by the cost of treatment. Supposed that the planner has set up the limited total cost of treatment for maintaining environmental quality as TOTCOST (Rp/day). This goal mathematically could be formulated as follow:

$$\sum_i \sum_j \sum_{k \in S_3} \psi_{ijk} X_{ijk} + \sum_i \sum_j \sum_{k \in S_2} \theta_i (X_{ijk}) \cong TOTCOST \qquad (16)$$

For all i, j, k.
Where:
S_3 = the set decision for treating residual to the each separate treatment system
S_2 = the set of decision for treating residual to the collective system these index of
ψ_{ijk} = unit cost of treatment system of point source- j in zone i and relating to alternative treatment system strategy k
θ_i = unit cost of collective treatment system in zone i for point source pollutant relating to alternative strategy k

Constraint System of Model

These constraints are important to assure that physical flows within the treatment system is balanced. Due to the system model, these constraints are not fully fuzziness content.

Total Pollutant Load in Points Sources Pollution

The total pollutant load from each point source must equal the subsequent flows in the system namely the pollutant load to be processed in primary treatment and amount pollutant to be discharged into the receiving points directly.

$$X_{ij1} + X_{ij2} = BOD_i^i \quad for\ all\ i\ and\ j \qquad (17)$$

Where:
η_{ijk}^p = estimated the total pollutant at point source j and zone i (Kg-BOD/day).

Residual Balance in Primary Treatment.

These constraints stipulate that effluent from the primary treatment must equal to the amount of pollutant removed in secondary treatment, discharged to the collective treatment system and to the receiving points, i.e.

$$(1-\eta_{ijk}^p) X_{ij1} - X_{ij4} - X_{ij3} - X_{ij5} = 0, \qquad (18)$$

Where:
η_{ijk}^p = removal efficiency of primary treatment of pollutant source j in zone i (%).

Residual Balance in Secondary Treatment System

These constraints indicate that effluent from the secondary treatment is equal with pollutant to be discharged directly to the receiving points and to be processed in the collective treatment system. To ensure this residual balance of flows, the following rigid constraints are necessary:

$$(1-\eta_{ijk}^s) X_{ij3} - X_{ij6} - X_{ij7} = 0,\ for\ all\ i\ and\ j \ (19)$$

Where:
η_{ijk}^s = removal efficiency of secondary treatment

Formulation Problem in The Fuzzy Goal Programming Model

A general water quality management in a river system is considered for developing the Fuzzy Goal Programming based on decision model developed in crisp equations. Referred to main goal of the environmental management in river system i.e.: environmental and economic goals that are associated with imprecise system, this model can be reformulated into Fuzzy Goal Programming model. This could be formulated in Linear Programming on crisp value as follows:

$$\text{Maximize } V(\mu) = \sum_{s=1} \mu_s \qquad (20)\text{-} (26)$$

Subject to:

$$\mu_i = \frac{U_i^{DO} - \sum_{i=1}^{m} a_{ri}\left\{\sum_{j=1}^{q_i}\sum_{k\in S_1} X_{ijk} + (1-\eta_i^c)\sum_{j=1}^{q_i}\sum_{k\in S_2} X_{ijk}\right\}}{U_i^{DO} - (C_i^s - STDO_i - NPS_i^{DO})} \quad \text{for } i = 1,2,\ldots m$$

$$\mu_i = \frac{U_i^{BOD} - \sum_{i=1}^{m} b_{ri}\left\{\sum_{j=1}^{q_i}\sum_{k\in S_1} X_{ijk} + (1-\eta_i^c)\sum_{j=1}^{q_i}\sum_{k\in S_2} X_{ijk}\right\}}{U_i^{BOD} - (C_i^s - STBOD_i - NPS_i^{BOD})} \quad \text{for } i = 1,,.m$$

$$\mu = \frac{U^{TotCost} - \sum_i\sum_j\sum_{k\in S_3}\psi_{ijk} X_{ijk} + \sum_i\sum_j\sum_{k\in S_2}\theta_i(X_{ijk})}{U^{TotCost} - TOTCOST} \quad \text{for } i = 1,\ldots m$$

$$X_{ij1} + X_{ij2} = BOD_j^i$$

$$(1-\eta_{ijk}^p)X_{ij1} - X_{ij4} - X_{ij3} - X_{ij5} = 0,$$

$$(1-\eta_{ijk}^s)X_{ij3} - X_{ij6} - X_{ij7} = 0$$

3. RESULTS AND DISCUSSION

In order to verify whether this Fuzzy Goal Programming model is workable or not, an output of solution model in a river case which was polluted organically is shown by a low DO level and increased BOD due to heavy organic pollutant discharge from major factories and residential area. This model provides information decisive about optimal pollutant allocation to be removed along the point sources in order to meet the management objectives.

U^s is the upper tolerance limit given for s objective functions for this model, that consists of objective achievement to close DO and BOD standards, and to achieve a least total treatment cost. i and j indices, other parameters or indices (i, j, k, S) and decision variables refer to previous equations.

In order to run the model, due to many variables and equations involved, the linear programming software standards for large linear program such as LINGO should be utilized. Equations (20) to (26) in crisp value as a conventional linier programming model that could be used for solving the fuzzy goal programming developed. Table 1 shows the existing DO and BOD levels in all zones. With respect to a set of desired standards, which DO level should be more than 4 mg/l and BOD should be lower than 6 mg/l, the solution model could make the objectives of the water quality management to be achieved. In order to reach these standards, the model give an optimal solution for minimizing the treatment cost of Rp 22.070.780,000 per day with taking into account the removal efficiency used in the primary, secondary and collective treatment are 30, 70 and 90% respectively.

Table 1 shows the optimal solution of model for residual allocation from the polluters into appropriate treatment system in meeting the desired target concentration of Standard Goal of DO ≥ 4 mg/l and BOD ≤ 6 mg/l for all zones. Actually, further improvement for some zones in terms of DO concentration could be more stringent than the first predetermined standards due actual water quality achieved after controlling better than this figure. Some industries must not be required to reduce the pollutant load, such as industries no 205, 206 and 317.

Table 1. Total Optimal Reduction Pollution Load Allocation at Each Alternative Waste Treatment System

Zone	Stream Quality Achieved (Before Controlling)) of DO (mg/l)	No of Polluter Industry	Total Pollutant Load (Kg-BOD/day)	Primary Treatment (Kg-BOD/day)	Direct to Body of Water (Kg-BOD/day)	Secondary Treatment (Kg-BOD/day)	Collective Treatment (Kg-BOD/day)
	4,0	101	63.32	63,32	-	44,32	13,30
		102	1359,60	565,37	-	395.,76	118,73
I		103	9,22	49,12	-	34,36	10,32
	4,4	204	95.56	95,66	-	66,96	20,09
		205	6777,81	-	221,41	-	-
		206	0,88	-	0,86	-	-
		207	1039,86	641,99	397,87	449,39	134,82
		208	5,21	2,28	2,93	1,60	0,48
		209	1579,18	406,157	1173,02	284,31	85,29
II		210	36,70	34,70	-	24,29	7,29
		211	150,09	74,27	85,42	51,99	15,60
		212	90,05	90,05	-	63,04	18,91
		213	61,38	61,38	-	42,97	12,89
		214	106,16	108,18	-	75,73	22,72

Zone	Stream Quality Achieved (Before Controlling)) of DO (mg/l)	No of Polluter Industry	Total Pollutant Load (Kg-BOD/day)	Primary Treatment (Kg-BOD/day)	Direct to Body of Water (Kg-BOD/day)	Secondary Treatment (Kg-BOD/day)	Collective Treatment (Kg-BOD/day)
III	3,1	315	833,89	833,89	-	583,72	175,12
		316	128,79	128,78	-	90,15	27,04
		317	2,71	-	3,17	-	-
		318	572,89	279,63	293,26	195,74	58,72
IV	3,9	419	394,20	394,20	-	275,94	82,78
		420	8,92	8,02	-	5,61	3,93
		421	31,80	9022853	22,76	6,32	1,89
		422	4,17	0,72	3,45	0,50	0,15
		423	421,06	421,06	-	294,76	88,43

4. CONCLUSION

A multiobjective Goal Programming Model for water quality management in river basin could be converted into Fuzzy Goal Programming model. The environmental management objectives included in the model are derived from set of objectives of environmental quality with BOD-DO parameters and economic objective with treatment cost parameter.The model generates a way of trade off among objectives in order to achieve the most desirable decision under given environmental quality and economic goals, and others. The next sensitivity analysis could be taken for the various combinations of desired stream standards associated to water utility as well as effluent standard for industries along the river. This approach provides an "optimistic" way to allocate pollutant removal along the river with appropriate treatment system and to determine the maximum allowable treatment cost, as well as which polluter should be controlled.

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